

The $\Delta\Delta$ Intermediate State in 1S_0 NN Scattering From Effective Field Theory

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We examine the role of the $\Delta\Delta$ intermediate state in NN scattering in the 1S_0 channel. The computation is performed at lowest order in an effective field theory involving local four-fermion operators and one pion exchange using dimensional regularization with \overline{MS} . As first discussed by Weinberg, in the theory with only nucleons the large scattering length in this channel requires a small scale for the local N^4 operators. When Δ 's are included (but without pions) a large scattering length can be obtained from operators with a scale $\sqrt{2M_N(M_\Delta - M_N)}$, but a fine tuning is required. The coefficients of the contact terms involving the Δ fields are not uniquely determined but for reasonable values one finds that in general NN scattering computed in the theory with Δ 's looks like that computed in the theory without Δ 's. The leading effect of the Δ 's is to change the coefficients of the four-nucleon contact terms between the theories with and without Δ 's. Further, the decoupling of the Δ 's in the limit of large mass and strong coupling is clearly demonstrated.

When pions are included the typical scale for the contact terms is $\sim 100\text{MeV}$, both with and without Δ 's and is not set by $\sqrt{2M_N(M_\Delta - M_N)}$. For reasonable values of contact terms that reproduce the scattering length and effective range (at lowest order) the phase shift is not well reproduced over a larger momentum range as is found in the theory without Δ 's at lowest order.

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1. Introduction

Several years ago Weinberg [1] suggested that low-energy nucleon-nucleon scattering and the interaction of multiple nucleons ought to be described by an effective field theory with a systematic expansion in powers of external momentum and the light quark masses. For NN scattering at momentum transfers much below the mass of the pion the theory is simply that of four-nucleon operators and at leading order in the expansion one finds that two such operators are required to describe the S-wave. The coefficients of these operators are related to the scattering lengths in the two spin channels ($S = 0$ and $S = 1$). These scattering lengths are much larger than what one might expect from the scale of strong interactions due to the close proximity to threshold of a scattering state in the $S = 0$ channel and the deuteron in the $S = 1$ channel. Hence, the scales of the coefficients of the operators in the effective theory are much smaller than the QCD scale. Since Weinberg's work there has been some substantial investigations that pursue this approach to nucleon interactions with the inclusion of pions, operators involving the quark masses and operators higher order in the momentum expansion [2] [3]. Further, inelastic processes have been considered such as pion-production in NN interactions [4].

It was recently shown how to compute the 1S_0 NN scattering amplitude and phase shift including pion interactions in dimensional regularization using \overline{MS} to renormalize the theory [3], with explicit computations performed at leading and subleading order. Without pions the leading order operators in the theory correspond to a zero range interaction and as a result the effective range vanishes. A non-zero effective range is obtained at next order in the expansion and the coefficient of the operator is fit to reproduce the experimental value of $r_{\text{eff.}} = 2.73\text{fm}$. When pions are included in the theory they alone contribute 1.3fm to the effective range, the rest being made up by a higher dimension operator. Further, the scale of the coefficients of the local four-nucleon operators in the theory with pions is substantially larger than in the theory without pions. (For comments on this approach see [5]).

The ultimate goal of this line of investigation is to describe NN scattering and inelastic processes involving nucleons in a theory with systematic power counting. This means that at a given order in the expansion parameter(s) an observable can be determined with a

given accuracy and that contributions from higher orders will be parametrically smaller. One suspects that in order to achieve this goal the low-lying baryon resonances may need to be incorporated as we expect the mass difference between the NN and resonance states will set the scale of the higher dimension operators. For S-wave scattering, the lowest excited configuration that can appear involves the $N^*(1440)$ and a nucleon. Unfortunately, the four baryon operators involving this resonance are unknown and are unrelated to quantities we know by any (approximate) global symmetry. The next excitation relevant for S-wave scattering is the $\Delta\Delta$ intermediate state. Again the four-baryon contact terms have unknown coefficients, but the Δ couplings to pions are reasonably well known (satisfying the $SU(4)$ spin-flavor relations at the 10% level, consistent with expectations of large N_c QCD). As we are performing a somewhat exploratory look at the effects of higher mass states we will work with the $\Delta\Delta$ intermediate state. Based on spin and isospin considerations alone there are 18 four-baryon local operators at lowest order required to describe the interactions between Δ 's and N 's. However, as we are considering NN scattering in the 1S_0 channel there are only 3 operators that can contribute. The lagrange density for the contact terms alone is given by

$$\mathcal{L} = -C^{NN}\mathcal{O}^{NN} - C^{\Delta N}\mathcal{O}^{\Delta N} - C^{\Delta\Delta}\mathcal{O}^{\Delta\Delta} \quad , \quad (1.1)$$

where

$$\begin{aligned} \mathcal{O}^{NN} &= \left[\frac{1}{4} \epsilon_{ij} \left(N_a^{\dagger j} N_b^{\dagger i} + a \leftrightarrow b \right) \right] \left[\frac{1}{4} \epsilon^{lm} \left(N_l^a N_m^b + a \leftrightarrow b \right) \right] \\ \mathcal{O}^{\Delta N} &= \left[\frac{1}{4} \epsilon_{ij} \left(N_c^{\dagger j} N_{c'}^{\dagger i} + c \leftrightarrow c' \right) \right] \left[\frac{3}{8\sqrt{5}} \epsilon^{ii'} \epsilon^{jj'} \epsilon^{kk'} \epsilon_{aa'} \epsilon_{bb'} \left(\Delta_{i'j'k'}^{a'b'c'} \Delta_{ijk}^{abc} + c \leftrightarrow c' \right) \right] \\ \mathcal{O}^{\Delta\Delta} &= \left[\frac{3}{8\sqrt{5}} \epsilon_{ii'} \epsilon_{jj'} \epsilon_{kk'} \epsilon^{aa'} \epsilon^{bb'} \left(\Delta_{abc}^{\dagger ijk} \Delta_{a'b'c'}^{\dagger i'j'k'} + c \leftrightarrow c' \right) \right] \\ &\quad \left[\frac{3}{8\sqrt{5}} \epsilon^{ll'} \epsilon^{mm'} \epsilon^{nn'} \epsilon_{dd'} \epsilon_{ee'} \left(\Delta_{l'm'n'}^{d'e'c'} \Delta_{lmn}^{dec} + c \leftrightarrow c' \right) \right] \end{aligned} \quad (1.2)$$

The annihilation operator Δ_{lmn}^{cef} for the Δ field is symmetric on its upper flavor indices, each running over $c, e, f = 1, 2$, and is symmetric on its lower spin indices which run over $l, m, n = 1, 2$. Similarly, for the nucleon field N_i^a , the upper flavor index runs over $a = 1, 2$ and the lower spin index runs over $i = 1, 2$. We have written the operators this way,

including their constants so that

$${}_{a'b'}\langle B'B'; S=0 | \mathcal{O}^{BB'} | BB; S=0 \rangle^{ab} = \frac{1}{2} (\delta_{a'}^a \delta_{b'}^b + \delta_{b'}^a \delta_{a'}^b) \quad , \quad (1.3)$$

where a, b and $a'b'$ are isospin indices while $B, B' = N$ or Δ . We see that three independent low energy observables are required to be measured in order to fix the lowest order constants $C^{NN}, C^{\Delta N}$ and $C^{\Delta\Delta}$. Such a requirement is clearly less than desirable, providing a limitation on the predictive power of the theory when Δ 's are included dynamically, and the situation becomes worse when we consider higher dimension operators. If we are willing to enforce the SU(4) spin-flavor symmetry that becomes exact in the limit of large N_c , then the three unknown C 's are reduced to two [6]. How well this symmetry describes the contact interactions of the Δ is still unknown and so we will not use the relations between the C 's in this work.

At leading order in the momentum expansion there are also contributions from pion exchange. The lagrange density describing such interactions is

$$\mathcal{L} = -g_A N^\dagger \sigma \cdot A N - g^{\Delta\Delta} \Delta^\dagger \sigma \cdot A \Delta - g^{\Delta N} (N^\dagger \sigma \cdot A \Delta + h.c.) \quad , \quad (1.4)$$

where A_k is the axial-vector field of pions

$$\begin{aligned} A_k &= \frac{\partial_k \Pi}{f_\pi} \\ \Pi &= \frac{1}{\sqrt{2}} \Pi^\alpha \tau^\alpha \end{aligned} \quad , \quad (1.5)$$

where τ^α are the Pauli matrices. The coupling $g_A = 1.25$ is well measured from β -decay, while the other constants are inferred from the strong decay of the Δ [7] [8]. They are found to agree well with the SU(4) spin-flavour symmetry relations

$$\frac{g^{\Delta\Delta}}{g_A} = \frac{9}{5} \quad , \quad \frac{g^{\Delta N}}{g_A} = \frac{3\sqrt{2}}{5} \quad . \quad (1.6)$$

The mass difference between the $\Delta\Delta$ and NN states is quite large on the scale of strong interactions, as we mentioned before, and it probably makes little sense to consider a theory with the Δ 's but without the pions. However, the ladder sum of one-pion exchanges can be made arbitrarily small by allowing the axial couplings to become small. In this

case the leading contribution to the NN scattering amplitude is the sum of bubbles arising from insertions of the local four-baryon operators. We will consider this scenario first as it allows a simple, analytical look at the inclusion of resonances. We then go on to consider the theory with the true values of the axial coupling constants.

2. The $g_A = g^{\Delta N} = g^{\Delta\Delta} = 0$ World

In the limit of vanishing axial couplings the leading contribution to the NN scattering amplitude is from the sum of all possible chains of four-baryon contact terms. These can be summed explicitly, as was shown by Weinberg to be the case for the theory of nucleons alone. It is convenient to write the amplitude for the process as

$$i\mathcal{A}_0 = \frac{-iC_E^*}{1 - C_E^* G_E^{NN}(\mathbf{0}, \mathbf{0})} \quad , \quad (2.1)$$

where it is straightforward to show that

$$C_E^* = C^{NN} + \frac{(C^{\Delta N})^2 G_E^{\Delta\Delta}(\mathbf{0}, \mathbf{0})}{1 - C^{\Delta\Delta} G_E^{\Delta\Delta}(\mathbf{0}, \mathbf{0})} \quad . \quad (2.2)$$

$G_E^{NN}(\mathbf{0}, \mathbf{0})$ and $G_E^{\Delta\Delta}(\mathbf{0}, \mathbf{0})$ are the $\mathbf{r} = \mathbf{0}$ to $\mathbf{r} = \mathbf{0}$ free green functions for the NN state and the $\Delta\Delta$ state of energy E respectively. In dimensional regularization they are

$$\begin{aligned} G_E^{NN}(\mathbf{0}, \mathbf{0}) &= \int \frac{d^d \mathbf{q}}{(2\pi)^d} \frac{1}{E - \mathbf{q}^2/M_N + i\epsilon} \\ &= -i \frac{M_N |\mathbf{p}|}{4\pi} + \mathcal{O}(d-3) \quad , \\ G_E^{\Delta\Delta}(\mathbf{0}, \mathbf{0}) &= \int \frac{d^d \mathbf{q}}{(2\pi)^d} \frac{1}{E - \mathbf{q}^2/M_N - 2\delta + i\epsilon} \\ &= \frac{M_N}{4\pi} \sqrt{2M_N \delta - |\mathbf{p}|^2 - i\epsilon} + \mathcal{O}(d-3) \end{aligned} \quad . \quad (2.3)$$

$\delta = M_\Delta - M_N$ is the Δ -nucleon mass difference and for the two nucleon system we have that $|\mathbf{p}|^2 = M_N E$. It is straightforward to derive analytic expressions for the scattering length and effective range in this theory (recall that $p \cot \delta = \frac{4\pi}{M_N} \text{Re}(1/\mathcal{A}) = -\frac{1}{a} + \frac{1}{2} r_{\text{eff}} |\mathbf{p}|^2 + \dots$),

$$\begin{aligned} a &= \frac{M_N}{4\pi} \frac{C^{NN} + ((C^{\Delta N})^2 - C^{\Delta\Delta} C^{NN}) G_{E=0}^{\Delta\Delta}(\mathbf{0}, \mathbf{0})}{1 - C^{\Delta\Delta} G_{E=0}^{\Delta\Delta}(\mathbf{0}, \mathbf{0})} \\ r_{\text{eff}} &= -\frac{M_N (C^{\Delta N})^2}{a^2 4\pi G_{E=0}^{\Delta\Delta}(\mathbf{0}, \mathbf{0})} \left[\frac{M_N}{4\pi} \frac{1}{C^{\Delta\Delta} G_{E=0}^{\Delta\Delta}(\mathbf{0}, \mathbf{0}) - 1} \right]^2 \quad . \end{aligned} \quad (2.4)$$

Notice that while the scattering length can have either sign, the effective range is negative definite.

In the case of $C^{\Delta N} = 0$ we return to the dynamics discussed by Weinberg where in order to reproduce the large scattering length in this channel of $a_0 = -23.7\text{fm}$, the coefficient $C_E^* = C^{NN} = 4\pi a_0/M_N = -1/(25\text{MeV})^2$ is much larger than one would have guessed from the scale of strong interactions alone. Unfortunately, we do not know what the coefficients C^{NN} , $C^{\Delta N}$ and $C^{\Delta\Delta}$ are, however, it would be unnatural for them to differ by orders of magnitude. One can see from (2.2) that if

$$C^{\Delta\Delta} \sim [G_{E=0}^{\Delta\Delta}(\mathbf{0}, \mathbf{0})]^{-1} \quad , \quad (2.5)$$

then $C_{E=0}^*$ is large. Hence for coefficients that are of a size one would expect from the scale of strong interactions the effective coupling constant of the N^4 -operator could be significantly larger than expected, due to a delicate cancellation. Such a precise cancellation is certainly not unnatural in the sense that in order to find a large scattering length compared to any physical length scale a cancellation of some sort is almost certainly required. Also, note that for a large value of $C^{\Delta\Delta}$ we return to the familiar scenario discussed by Weinberg, where the theory with Δ 's is identical to the theory without Δ 's. In fact, for reasonable values such as $C^{\Delta\Delta} \sim 1/(100\text{ MeV})^2$ which gives $C^{\Delta\Delta} G_{E=0}^{\Delta\Delta}(\mathbf{0}, \mathbf{0}) \sim 5.5$ the theory behaves largely like the theory of nucleons alone.

It is illuminating to consider special cases for the couplings that reproduce the scattering length in this channel. For $C^{NN} = C^{\Delta N} = C^{\Delta\Delta} = C_0$ we find from (2.4) that

$$C_0 = \frac{4\pi}{M_N} \frac{1}{\left(\frac{1}{a_0} + \sqrt{2M_N\delta}\right)} \quad . \quad (2.6)$$

$$\sim + \frac{1}{(234\text{MeV})^2}$$

The inclusion of the Δ 's has introduced an intrinsic length scale $\sqrt{2M_N\delta}$, which now sets the scale for the coefficients of the four-baryon operators. More importantly, we see that for a large scattering length, such as is found in this scattering channel, the coefficients must be finely tuned so that there is substantial cancellation between $1/C_0$ and $G_{E=0}^{\Delta\Delta}(\mathbf{0}, \mathbf{0})$ in (2.1) as discussed above. The effective range for this parameter set is $r_{\text{eff.}} \sim -0.26\text{fm}$, which is

to be compared to the measured value of $r_{\text{eff.}} \sim +2.73\text{fm}$. For another choice of coefficients, $C^{NN} = \frac{1}{2}C^{\Delta N} = \frac{1}{4}C^{\Delta\Delta} = C_0$, a fit to the scattering length yields $C_0 \sim +\frac{1}{(470\text{MeV})^2}$ and in turn we find the effective range to be $r_{\text{eff.}} \sim -1.05\text{fm}$. We see in these two examples the problem of having large coefficients of the four-nucleon operators is traded for a fine tuning problem when the Δ 's are included. An analogous situation would arise if one was to regulate the nucleon theory alone with a momentum cut-off procedure ² where the scattering length results from a fine tuning between the cutoff and coefficients in the expansion.

It is simple to see from (2.1), (2.2) and (2.4) how the Δ 's decouple from the theory as they become much more massive than the nucleon. As $\delta \rightarrow \infty$ the green function $G_E^{\Delta\Delta}(\mathbf{0}, \mathbf{0})$ becomes large and we can expand C_E^* in inverse powers of $G_E^{\Delta\Delta}(\mathbf{0}, \mathbf{0})$,

$$C_E^* = C^{NN} - \frac{(C^{\Delta N})^2}{C^{\Delta\Delta}} - \frac{4\pi}{M_N \sqrt{2M_N \delta - |\mathbf{p}|^2}} \left(\frac{C^{\Delta N}}{C^{\Delta\Delta}} \right)^2 + \dots \quad (2.7)$$

This tends to a momentum independent constant as $\delta \rightarrow \infty$, and we recover Weinbergs summation of nucleon bubbles, with a coefficient

$$C = C^{NN} - \frac{(C^{\Delta N})^2}{C^{\Delta\Delta}} \quad (2.8)$$

Expanding $p \cot(\delta)$, the scattering length and effective range in (2.4) become

$$\begin{aligned} a &\rightarrow \frac{M_N}{4\pi} \left[C^{NN} - \frac{(C^{\Delta N})^2}{C^{\Delta\Delta}} \right] - \left(\frac{C^{\Delta N}}{C^{\Delta\Delta}} \right)^2 \frac{1}{\sqrt{2M_N \delta}} + \dots \\ r_{\text{eff}} &\rightarrow - \left(\frac{C^{\Delta N}}{C^{\Delta\Delta}} \right)^2 \frac{1}{a^2} \frac{1}{(2M_N \delta)^{\frac{3}{2}}} + \dots \end{aligned} \quad (2.9)$$

One should note that (2.8) demonstrates that the coupling of the N^4 operator in the theory of nucleons alone is not directly related to the coupling of the N^4 operator in the theory including Δ 's. This means that a direct comparison between extractions of C_E^* in (2.2) from nuclei or NN interactions with predictions or relations from QCD (e.g. [6]) is not straightforward.

² We thank Peter Lepage for discussions on this point

3. The Real World: Non-Zero Axial Couplings.

In the previous section we discussed a theory with vanishing axial couplings. Although not directly related to nature, it did demonstrate interesting behaviour, particularly in how the Δ 's decouple from the nucleon sector. In this section we include leading order pion exchange into the system, and extend the work of [3]. The procedure for the inclusion of pions is essentially that discussed in [3] with the additional complication of including the $\Delta\Delta$ channel and so we will only present an outline of the procedure.

The calculation of the phase shift in the 1S_0 channel is split up into two parts. The first part involves computing the amplitude from the yukawa ladder sum arising from single pion exchange $i\mathcal{A}_\pi$ where there are both NN and $\Delta\Delta$ intermediate states, see fig. 1. The second part is computing the amplitude from the sum of all possible insertions of the local contact terms dressed with yukawa ladder exchanges $i\mathcal{A}_{\text{bub.}}$, see fig. 2. The total amplitude is the sum of these two parts

$$i\mathcal{A} = i\mathcal{A}_\pi + i\mathcal{A}_{\text{bub.}} \quad . \quad (3.1)$$

To compute the yukawa ladder sum for $NN \rightarrow NN$ scattering one solves the schrodinger equation for the wavefunctions Ψ_E^N and Ψ_E^Δ . Defining $U^N = r\Psi_E^N$ and $U^\Delta = r\Psi_E^\Delta$, we solve

$$\left[-\frac{1}{M_N} \frac{d^2}{dr^2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{e^{-m_\pi r}}{r} \begin{pmatrix} \alpha^{NN} & \alpha^{\Delta N} \\ \alpha^{\Delta N} & \alpha^{\Delta\Delta} \end{pmatrix} - \begin{pmatrix} E & 0 \\ 0 & E - 2\delta \end{pmatrix} \right] \begin{pmatrix} U^N \\ U^\Delta \end{pmatrix} = 0 \quad , \quad (3.2)$$

subject to the condition that $U^\Delta \rightarrow e^{-r\sqrt{2M_N\delta - |\mathbf{p}|^2}}$ asymptotically (as we are working below the energy required for $\Delta\Delta$ production). The NN wavefunction obtained as a solution to (3.2) is compared with the asymptotic behaviour

$$\Psi_E^N(r) \xrightarrow{r \rightarrow \infty} -\frac{i}{2} \left(e^{2i\delta_\pi} \frac{e^{ipr}}{pr} - \frac{e^{-ipr}}{pr} \right) \quad , \quad (3.3)$$

for an s-wave, to obtain the phase shift from pion exchange alone, δ_π . The yukawa couplings in (3.2) are related to the couplings appearing in (1.4) by

$$\begin{aligned} \alpha^{NN} &= \frac{g_A^2 m_\pi^2}{8\pi f_\pi^2} \\ \alpha^{\Delta N} &= \frac{\sqrt{5}(g^{\Delta N})^2 m_\pi^2}{9\pi f_\pi^2} \\ \alpha^{\Delta\Delta} &= \frac{11(g^{\Delta\Delta})^2 m_\pi^2}{72\pi f_\pi^2} \end{aligned} \quad . \quad (3.4)$$

The amplitude $i\mathcal{A}_\pi$ can be found directly from the phase shift

$$i\mathcal{A}_\pi = \frac{4\pi}{2M_{Np}} (e^{2i\delta_\pi} - 1) \quad . \quad (3.5)$$

The amplitude resulting from multiple insertions of the contact operators can be found straightforwardly by explicit computation of the bubble chains. The coefficients that appear in the bubble chain are modified from those that appear in (1.1) due to a contribution from the part of the pion potential of the form $\delta^3(\mathbf{r})$. As in [3] we use a tilde to denote the modified coefficients

$$\begin{aligned} \tilde{C}^{NN} &= C^{NN} + \frac{g_A^2}{2f_\pi^2} \\ \tilde{C}^{\Delta N} &= C^{\Delta N} + \frac{20}{9\sqrt{5}} \frac{(g^{\Delta N})^2}{f_\pi^2} \quad . \\ \tilde{C}^{\Delta\Delta} &= C^{\Delta\Delta} + \frac{11}{18} \frac{(g^{\Delta\Delta})^2}{f_\pi^2} \end{aligned} \quad (3.6)$$

Defining

$$\begin{aligned} X &= 1 - \tilde{C}^{NN} G_E^{NN, \overline{MS}}(\mathbf{0}, \mathbf{0}) \\ Y &= 1 - \tilde{C}^{\Delta\Delta} G_E^{\Delta\Delta, \overline{MS}}(\mathbf{0}, \mathbf{0}) \\ Z &= \tilde{C}^{\Delta N} G_E^{\Delta\Delta, \overline{MS}}(\mathbf{0}, \mathbf{0}) + \tilde{C}^{NN} G_E^{\Delta N, \overline{MS}}(\mathbf{0}, \mathbf{0}) \quad , \\ W &= \tilde{C}^{\Delta N} G_E^{NN, \overline{MS}}(\mathbf{0}, \mathbf{0}) + \tilde{C}^{\Delta\Delta} G_E^{\Delta N, \overline{MS}}(\mathbf{0}, \mathbf{0}) \end{aligned} \quad (3.7)$$

the bubble sum is

$$\begin{aligned} iA_{\text{bub.}} &= -i \left[\frac{[\Psi_E^N(0)]^2}{X} \left[\tilde{C}^{NN} + \tilde{C}^{\Delta N} \frac{Z}{XY - ZW} \right] \right. \\ &\quad + 2 \frac{\Psi_E^N(0) \Psi_E^\Delta(0) \tilde{C}^{\Delta N}}{XY - ZW} \quad . \\ &\quad \left. + \frac{[\Psi_E^\Delta(0)]^2}{Y} \left[\tilde{C}^{\Delta\Delta} + \tilde{C}^{\Delta N} \frac{W}{XY - ZW} \right] \right] \end{aligned} \quad (3.8)$$

The functions $\Psi_E^N(0)$ and $\Psi_E^\Delta(0)$ are determined by (3.2) and $G_E^{ij, \overline{MS}}(\mathbf{0}, \mathbf{0})$ are green functions regulated in \overline{MS} . We compute them numerically by finding solutions to

$$\begin{aligned} \left[-\frac{1}{M_N} \nabla^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{e^{-m_\pi r}}{r} \begin{pmatrix} \alpha^{NN} & \alpha^{\Delta N} \\ \alpha^{\Delta N} & \alpha^{\Delta\Delta} \end{pmatrix} - \begin{pmatrix} E & 0 \\ 0 & E - 2\delta \end{pmatrix} \right] \begin{pmatrix} G_E^{NN} \\ G_E^{\Delta N} \end{pmatrix} = \delta^3(\mathbf{r}) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ \left[-\frac{1}{M_N} \nabla^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{e^{-m_\pi r}}{r} \begin{pmatrix} \alpha^{NN} & \alpha^{\Delta N} \\ \alpha^{\Delta N} & \alpha^{\Delta\Delta} \end{pmatrix} - \begin{pmatrix} E & 0 \\ 0 & E - 2\delta \end{pmatrix} \right] \begin{pmatrix} G_E^{\Delta N} \\ G_E^{\Delta\Delta} \end{pmatrix} = \delta^3(\mathbf{r}) \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad . \end{aligned} \quad (3.9)$$

The boundary conditions are that G_E^{NN} and $G_E^{\Delta N}$ have no incoming wave and that $G_E^{N\Delta}$ and $G_E^{\Delta\Delta}$ fall exponentially at large radius. We have used shorthand for $G_E^{ij} = G_E^{ij}(\mathbf{r}, \mathbf{0})$. The $G_E^{ij}(\mathbf{0}, \mathbf{0})$ are divergent as computed from (3.9) and are regulated using dimensional regularization with modified minimal subtraction \overline{MS} , in analogy with the procedure of [3]. One finds that the green functions defined in \overline{MS} are simply related to those determined in (3.9) by

$$G_E^{ij, \overline{MS}}(\mathbf{0}, \mathbf{0}) = \lim_{r \rightarrow 0} \left[G_E^{ij}(\mathbf{r}, \mathbf{0}) + \frac{M_N}{4\pi r} \delta^{ij} - \frac{\alpha^{ij} M_N^2}{8\pi} [2 \log(\mu r) + 2\gamma - 1] \right] . \quad (3.10)$$

The constants C^{ij} are defined in \overline{MS} by this procedure and must be fit to data.

We have the leading order amplitude defined in terms of the three coefficients of the local contact terms C^{ij} and the three axial coupling constants g^{ij} . As mentioned previously, the axial couplings constants are found to obey the SU(4) relations rather well and we will use these relations for the remaining discussion. We discuss results obtained for some choices of the coefficients of the contact terms but are unable to say anything more concrete. By construction, the low momentum behaviour of the phase shift is the same for all choices of C^{ij} that reproduce the scattering length, whose structure is dominated at very low momentum by $\cot(\delta) = -1/(ka_0)$. One finds that deviations in the phase shift between different choices of C^{ij} arise around 15MeV. This comes as no surprise as we know that a higher dimension operator with a scale of $\sim 100\text{MeV}$ is required to reproduce the effective range in the absence of Δ 's and that, in general, the Δ 's themselves do not reproduce the effective range.

Lets us again consider the case of $C^{NN} = C^{\Delta N} = C^{\Delta\Delta} = C_0$ where fitting to the scattering length requires $C_0 = -1/(106.2\text{MeV})^2$. The effective range predicted by this choice of parameters is $r_{\text{eff.}} = 4.3\text{fm}$. However, despite this reasonably good fit to these two low energy parameters, the phase shift over a larger momentum interval, below $|\mathbf{p}| < 150\text{MeV}$ is relatively poor. The same can be said for a choice of parameters that reproduces both the scattering length and the effective range, $C^{\Delta\Delta}/1.17 = C^{\Delta N}/1.17 = C^{NN} = -1/(110.995\text{MeV})^2$, as can be seen in fig. 3. In fact, we find that the theory with Δ 's is little different from the theory without Δ 's, and that they have effectively decoupled. The inclusion of the Δ 's does not really improve the situation for reasonable

size four-baryon operators.

A discussion of an "optimal" expansion to perform in order to reproduce the observed phase shift can be found in [3]. An expansion of $k \cot(\delta)$ in insertions of higher dimension operators was found to be more rapidly converging than an expansion of the amplitude itself. While such a procedure is simple for the inclusion of higher dimension N^4 operators, it is not clear how to perform this for the inclusion of the Δ 's, and other baryon resonances. There is no expansion parameter to expand in when the Δ 's and other resonances are included at lowest order. Therefore, we should have expected all along to be able to do little better than we were able to do in the theory with nucleons alone. This is precisely what we found. Recently, it was suggested [9] that the inclusion of an explicit dibaryon field in channels with large scattering lengths allows one to recover "reasonable" power counting. In such a scheme the effects of the $\Delta\Delta$ interactions could be included perturbatively and could be seen to decouple in a more transparent manner.

4. Conclusions

We have included the $\Delta\Delta$ intermediate state into the calculation of NN scattering in the 1S_0 channel using the methods of [1] and [3]. In the limit of vanishing axial couplings of the pions to the N 's and Δ 's we find that the large scattering length can arise from coefficients of natural size (scale set by $\sqrt{2M_N\delta}$) but a fine tuning is required. Further, we can explicitly see how the Δ 's decouple from the theory as either the $\Delta - N$ mass difference becomes large or as the Δ^4 interactions become strong. For realistic values of the contact terms (presently unknown) the leading order NN scattering amplitude in the theory with Δ 's closely resembles that of a theory with no Δ 's.

When the axial couplings are reinstated one finds that the large scattering length is reproduced by contact terms with a scale ~ 100 MeV. One also can obtain effective ranges of the correct size for particular values of the C^{ij} without the inclusion of higher dimension operators. Generally, one fails to reproduce the phase shift over larger momentum intervals at leading order in exactly the same way one fails to do so with nucleons alone as the $\Delta\Delta$ state has effectively decoupled from the theory. Its leading effect is to modify the value

of the coefficients of the four-nucleon contact terms. A higher order contact term is still required to extend the range of momentum over which the effective theory reproduces the data.

The coefficients of the local operators are not presently determined by experiment, although two combinations are constrained at leading order by the scattering length and effective range (assuming $SU(4)$ symmetry for the axial couplings). This ignorance severely restricts the depth to which we can investigate the role of Δ 's in NN scattering.

As we mentioned previously, the $\Delta\Delta$ intermediate state is not the lightest excited state(s) that can appear in this channel. However, the fact that the Δ and the N together form an $SU(4)$ multiplet leads one to suspect that the interactions between the Δ 's and the N 's will be stronger than between these baryons and baryons outside the $SU(4)$ multiplet. Such an effect is evidenced in the coupling between pions and nucleons with the Δ and with other higher mass resonances such as the $N^*(1440)$. It is for this reason that we have examined the $\Delta\Delta$ intermediate state. However, we have learned from the limit of vanishing axial couplings that the stronger the interaction, the less effect the state has on the overall scattering, i.e. in the limit of large $C^{\Delta\Delta}$ we return to the theory of NN interactions alone at leading order. It may turn out to be the case that the other intermediate states have a larger effect on the scattering because they have a weaker interaction.

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Figure Captions

- Fig. 1. The ladder sum of yukawa exchanges with both NN and $\Delta\Delta$ intermediate states.
- Fig. 2. Insertions of the contact terms dressed with yukawa exchanges denoted by the light shaded regions.
- Fig. 3. The 1S_0 phase shift at leading order in the expansion verses centre-of-mass momentum. The dashed curve is the the theory without Δ 's and the solid curve is $C^{NN} = C^{\Delta N}/1.17 = C^{\Delta\Delta}/1.17 \sim -1/(110\text{MeV})^2$ fit to reproduce the scattering length and effective range. The points correspond to the phase shift data from the Nijmegen partial wave analysis [10]

Fig. 1

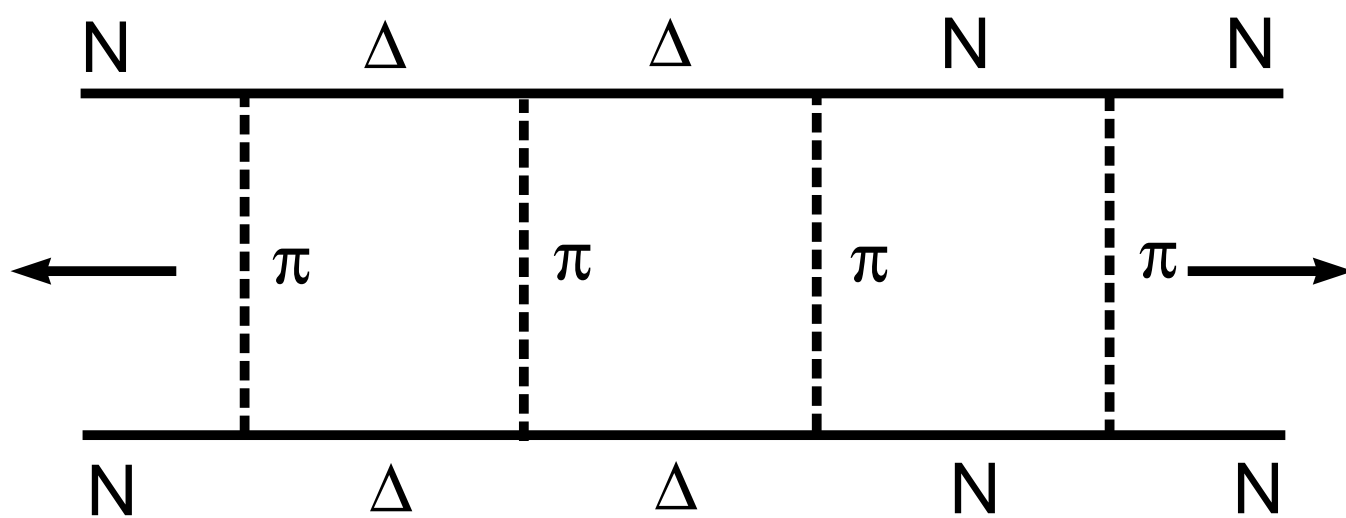


Fig. 2

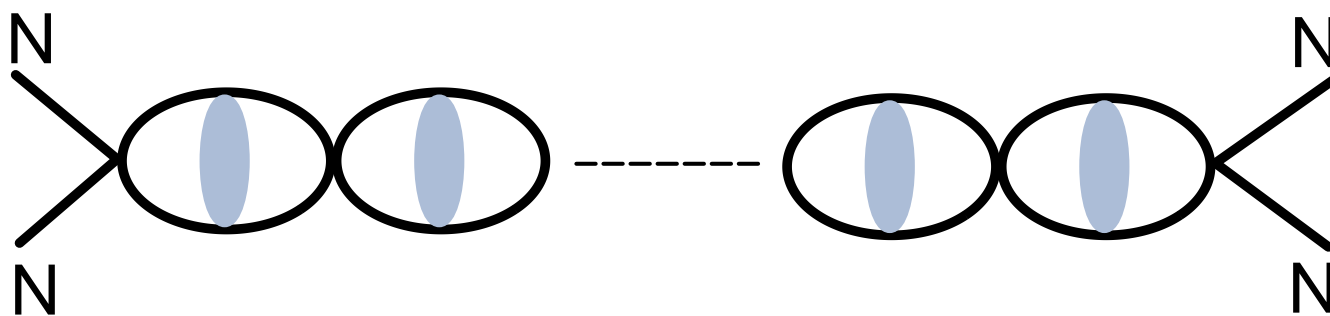
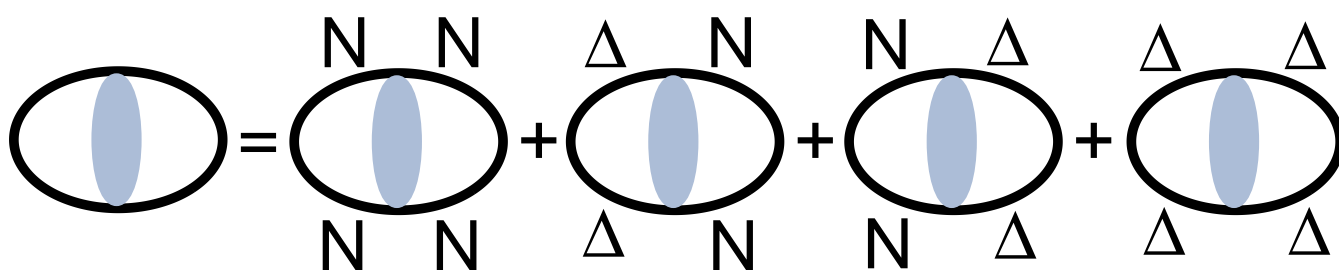


Fig. 3

